Disassortative degree mixing and fractality of scale-free networks

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From the viewpoint of the relation between the shortest path distance and the network size, we can classify most of the complex networks in the real world into two classes, namely, small-world and fractal networks [1,2]. It has been clarified that the small-world property originates from the existence of short-cut edges. On the other hand, the formation mechanism of fractal networks in the real world is still an open question. Nevertheless, we empirically know that fractal scale-free networks exhibit disassortative degree mixing [3]. Conversely, if disassortative mixing makes a scale-free network fractal, the origin of fractality would be found in disassortativity. At first sight, this hypothesis seems to be false, because the fractal property is a consequence of long-range structural correlations beyond the nearest neighbors, while disassortativity is a short-range property characterizing nearest neighbor degree correlations. It has been, however, demonstrated that even short-range disassortative degree mixing induces a long-range correlation if the network exhibits the scale-free property [4]. It is, thus, not obvious whether disassortativity is the origin of fractality in scale-free networks.

In this work, we examine the above possibility. To this end, we create maximally disassortative (MD) networks formed by rewiring edges starting from an initial uncorrelated scale-free network, while keeping the degree sequence. If the MD network possesses the fractal property, one can conclude that disassortative degree mixing makes scale-free networks fractal. Initial uncorrelated networks are formed by rewiring randomly edges of the (u, v)-flower [5]. As a result, there are many MD networks with different topologies, but most of them are not fractal. This result shows that disassortativity does not always induce fractality of scale-free networks. Also, we suggest that the fractal property requires a long-range repulsive correlation between similar degrees in scale-free networks.

[l] D.J. Watts and S.H. Strogatz, Nature **393**, 440 (1998).

[2] C. Song, et al., Nature **433**, 392 (2005).

[3] S.-H. Yook, et al., Phys. Rev. E **72**, 045105 (2005).

[4] J. Menche, et al., Phys. Rev. E **81**, 046103 (2010).

[5] H.D. Rozenfeld, et al., New J. Phys. **9**, 175 (2007).